Collaborative Problem Solving and Discovered Complexity

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Two collaborative groups of senior secondary mathematics students with similar ability to solve unfamiliar challenging problems demonstrated different levels of engagement and different levels of conceptual development when they worked with the same task. Study of the nature of task complexity led to the formulation of *discovered complexity* as a useful tool to analyse student response to tasks. It was found the task undertaken provided the opportunity for discovered complexities that possessed the potential to enhance student learning. Use of such a task was found insufficient to ensure discovered complexity resulted.

Introduction

As a teacher of senior secondary mathematics I gradually recognised my original teacher-centred approach catered inadequately for students in a mixed-ability mathematics classroom. Some students did not have access to the curriculum because they lacked the required background. In my opinion, too many students felt they were failures at mathematics and too few students were challenged and motivated by the mathematics presented. Over a period of years I developed my own collaborative approach (Class Collaboration explained in Table 2) and found students often demonstrated a high level of engagement with the task and articulated an increased understanding of mathematics. This collaborative approach to mathematics learning excited students of both genders:

> I just love maths now. It makes such a difference when you can explain what you understand to each other. I get so excited when we solve problems. I know I can now solve them by myself.
> (Year 11 female student)

> It is so exciting when you can report something you know no one else [no other group] has found.
> (Year 12 male student)

M. Csikszentmihalyi's (1992) concept of flow is consistent with the engagement I observed as students in some collaborative groups became so involved in the task that they lost all consciousness of time, self and the world around them. Naidra, a Year 11 student studying mathematics through Class Collaboration (Barnes, in Press), described breakthroughs or insights in mathematics as "magical". He claimed that such moments often occurred for him and that when they did it was "great". M. Csikszentmihalyi and I. Csikszentmihalyi (1992) found people in flow described such feelings of pleasure and exhilaration and that optimal learning conditions existed. Either individual or group flow could occur when people worked just above their present skill level with a challenge almost out of reach. The optimal learning conditions that accompany group flow are consistent with the increased conceptual development that can result when students collaborate in socio-cognitive settings (Bell, 1993; Brown, 1994; Cobb, Wood, Yackel, & McNeal, 1992).

As "No activity can sustain it [flow] for long unless both the challenges and skills become more complex" (Csikszentmihalyi and Csikszentmihalyi, 1992, p. 30), the nature of task complexity and how the complexity of the task might change are integral to studies of sustained student engagement. In an attempt to understand this link between task complexity and student engagement, I undertook research in my own classroom posing the question: What is the nature of task complexity and how do students respond to this complexity when problem solving is undertaken in a collaborative setting?

The term *complex* is frequently used to describe tasks (Schoenfeld, 1985; Tang, 1993) without explicit definition of *complex*. Where an implicit or explicit explanation of the nature of task complexity exists, task complexity is most frequently equated with intellectual complexity (Smith and Stein, 1998; Cohen, 1994; Tannenbaum, 1983). Advocates of the use of complex mathematical tasks for either instruction or assessment presently lack a structure (and a vocabulary) by which to identify and evaluate the different characteristics of such tasks and their relation to the sort of student activity they promote. An understanding of those aspects of task complexity that facilitate student engagement and enhance conceptual development will be of assistance in task construction, task selection and the implementation of curriculum related to such tasks. The present scarcity of definition in the literature raises questions about the usefulness of the term 'complexity' as utilised in such documentation as VCE Mathematics 2000 (Victorian Board of Studies, 1999).

Theoretical Constructs and Methodology

In this study, the term *unfamiliar challenging problems* refers to tasks that: (a) are presented before the relevant mathematical concepts have been 'taught'; (b) cannot be solved by the application of algorithmic procedures assumed known to the students; and (c) require students to analyse mathematical representations to connect mathematical ideas and to build concepts new to the them.

A distinction is made between a student's capacity to solve unfamiliar challenging problems (*Student Ability*) (Krutetskii, 1976) and the demonstrated problem solving achievements of students in a collaborative setting as they solve a problem together (Tang, 1993). Whether Student Ability is innate, dependent on previous problem solving experiences or a combination of such factors is immaterial to this study.

Table 1.
Theoretical Hierarchical Construct of Ability Formulated using Bloom's Taxonomy (1956) and Krutetskii (1976)

Categories of Ability	Hierarchy of mental activity activated (in descending order)
Evaluative-synthesis	Recognise inconsistent information
Synthesis	Combine two or more concepts to create an original concept
Analytic-synthesis	Explain the need for extra information Use more than one pathway
Analysis	Recognise the need for extra information Build on a concept to answer a question with a slight change
Comprehension	Understand a concept
Knowledge	Repeat a taught idea

To ascertain the relative Ability of the students in this study, a theoretical construct of Ability was formulated using a hierarchy of cognitive activities based on Bloom's Taxonomy (1956) and Krutetskii's (1976) empirical data. These cognitive activities are displayed in descending order in Table 1. Student capability with respect to this hierarchy was measured using a pen and paper test designed to measure Student Ability. These results informed my selection of collaborative groups with similar Student Ability distributions for case study. The construct of cognitive abilities developed to design this test focused my analysis of student response to the task.

Collaborative instruction operated at two levels within the classrooms studied. Each small group collaborated as they worked with the problem but the class as a whole also collaborated to share ideas at regular intervals. The main characteristics of the Class Collaboration approach are detailed in Table 2 and Williams (1997).

Table 2.
Features of the Class Collaboration Approach and Elaboration of these Features

Feature	Elaboration of this Feature
The learning culture	Build from each student's present understanding Any justified solution pathway is acceptable
Type of task	Open ended task undertaken over several lessons A maximum of ten minutes teaching prior to task
Classroom management	Small collaborative groups Regular feedback from each group to whole class
Questioning technique	As groups develop ideas, the teacher: Visits each group regularly for 30 seconds to 2 minutes; Listens to and sometimes participates in discussions; Demonstrates an interest in the students' thoughts; Asks questions to assist groups to clarify, refocus upon a productive pathway, experience an insight, analyse and evaluate progress; Shares the excitement when groups find new ideas; Does not provide hints nor indicate correctness of pathway.
Reporting process	Class members may ask the reporter for clarification. A group may discard, use or build upon ideas presented.

The nature of task complexity was explored through interviews with experts from the fields of Mathematics, Mathematics Education and Gifted Education (*Experts*). These Experts provided opinions about the relative complexity of two unfamiliar challenging problems. The purpose of this comparison was not to draw conclusions as to which task was more complex in any absolute sense, but rather to use both tasks as a mechanism for eliciting the opinions of these Experts about the nature of task complexity. Experts found the six dimensions of complexity within the Williams / Clarke Framework (linguistic, contextual, operational, conceptual, intellectual, and representational complexity) sufficient to identify the dimensions of complexity within the tasks (Williams & Clarke, 1997).

Although Experts differed in their opinions about the nature of task complexity all Experts included some aspect of Intellectual Complexity in their description (Williams & Clarke, 1997). *Discovered Complexity* emerged as Expert opinion was analysed. It is a major premise of this study that an analysis in terms of discovered complexities advances our understanding of both student collaborative activities and the function of mathematical tasks in promoting such activity. *Discovered complexities* are not apparent at the beginning of the task but become evident during task performance. These discovered complexities possess two key features that meet Csikszentmihalyi's (1992) stated conditions for flow; (a) students focus on a search to answer a question implicitly or explicitly formulated by the group (intellectual challenge); and (b) this search encompasses mathematical ideas and concepts new to all group members.

Prior to this research, I devised the task undertaken by the students in this study, 'Understanding the Double Derivative', to allow students to develop their own concept of why the second derivative in combination with the derivative gives information about the shape of the curve. A prerequisite for this task is the knowledge that the derivative of a function is the gradient of its graph. The task also assumes that students have done no prior work on second derivatives (Williams & Clarke, 1997).

The research subjects were students in two classes studying high level final year secondary mathematics (Specialist Mathematics) in a large Victorian metropolitan government school. These students had undertaken a Gradient Investigation prior to the research period and the Ability test at the commencement of the research period. The Gradient Investigation was intended to provide all students with a common background in the mathematical ideas prerequisite for both the Ability test and the second derivative task. Without such common background, valid inferences could not be drawn about students' cognitive activity as they responded to the Ability test.

Group 1 (Talei, William and Gerard) from Class 1 and Group 2 (Dean, Alistair, Tony and Rez) from Class 2 were selected for case study because these two groups had similar Ability distributions, but demonstrated different learning outcomes. On the Ability test, the majority of the members in each group demonstrated the ability to 'combine concepts to create an original concept'. William (Group 1) also demonstrated the higher level ability to 'recognise inconsistent information' and the highest cognitive activity demonstrated by Rez (Group 2) was the ability to 'use more than one pathway' (see Table 1).

Both groups undertook collaborative work and individual work, but in the opposite order. Changes in each student's level of mathematical understanding were monitored by tests of mathematical understanding administered before and after each instructional period. Students were video taped as they responded to the task during Class Collaboration. The mathematical focus of each discussion was identified and the time devoted to this topic recorded (Tables 3 and 4). I concluded that a student had improved his or her development of a concept if the understanding displayed in the video and the post-test of mathematical understanding, given after the collaborative work, was deeper than the understanding displayed on the pre-test of mathematical understanding (Skemp, 1976, 1979). To draw inferences about student engagement, relevant aspects of body language were considered (Quilliam, 1995) and the validity of my interpretations of body language was checked through discussion with four colleagues undertaking their PhD in Education.

Results

Group 1 (Talei, William and Gerard) undertook Class Collaboration before individual work. Table 3 summarises the nature and number of complexities discovered by Group 1 in the first thirteen minutes of collaborative group work on Task A and during the extra minute that student discourse continued after the prescribed time interval had ended.

Group 1 worked for short time intervals to develop each of many mathematical concepts as they resolved the focus questions related to a variety of discovered complexities. My explicit formulation of the spontaneous question associated with each discovered complexity, in Table 3, demonstrates the progressive development of conceptual understanding. As all three students participated in the initial discussions related to each discovered complexity, it can be assumed conceptual development was realised by each of the three students.

Table 3.
Group 1, Mathematical Content of First Collaborative Session (Talei, William, and Gerard)

Amount of Time	Mathematical Content (emergent in student dialogue)	Complexities and * Discovered Complexities
4 mins	Construction of gradient graph. Sign Diagram: meaning and construction.	Intellectual (analytic-synthesis), Conceptual, Representational.
2 mins	Return to problem to clarify. Analyse graphs and diagrams searching for patterns.	Linguistic, Intellectual (understanding, analysis).
1 min	What are the relative benefits of a sign diagram and a graph?	**Discovered Complexity*
	What characteristics of $f(x)$ are evident (shape) and are not evident (position) through analysis of $f''(x)$? Analysis through algebra of extra information required.	**Discovered Complexity*
2 mins	What do the critical features of $f''(x)$ indicate (inflections) and not indicate (turning points) about $f(x)$?	**Discovered Complexity*
1 min	Continue searching for patterns. $f''(x)$ shows inflection but not turning points. Why?	Intellectual(analysis) **Discovered Complexity*
2 mins	Further analysis of relative benefits of sign diagram and graph	**Discovered Complexity*
1 min	How much information is required to generate $f(x)$ from $f''(x)$?	**Discovered Complexity*
1 min	Why isn't the turning point of $f(x)$ a critical feature of the graph of $f''(x)$?	**Discovered Complexity*

Note. *All Discovered Complexities included Conceptual Complexities, Representational Complexities and higher level Intellectual Complexities.

In the minute of collaborative group work, after the buzzer, Talei and William (and initially Gerard) displayed a high level of engagement as they investigated the discovered complexity precipitated by William's statement that there "must be something more". William, Talei and Gerard did not respond to the buzzer but instead continued to lean forward over the page and build upon each other's comments. The three students did not appear to hear the teacher providing instructions about the reporting session. William and Talei did not appear to hear Gerard who had turned eventually to listen to the teacher and then turned to ask Talei and William a question about his forthcoming report for the group. After further brainstorming with Talei, William exclaimed—with the accompanying hand movement tracing a minimum point in the air—"it is always turning the same way". The video record captured the instant where William suddenly appeared to grasp the overall concept. His utterance "That's it!" and the accompanying hand movement together with the apparent expression of enlightenment on his face indicated something had suddenly become clear. Talei's hand movement at almost the same instant indicated she had also grasped the concept. These students were in flow as the concept was constructed and a "jolt of thrills" was evident in William's exclamation (Sato, 1992, p. 92).

Table 4.
Group 2, Mathematical Content of First Collaborative Session (Rez, Alistair, Dean and Tony)

Amount of Time	Mathematical Content (emergent in student dialogue)	Complexities and * Discovered Complexities
4 mins	Derivatives investigated and generalised algebraically .	Numerical , Intellectual (comprehension, analysis)
5 mins	Comparison of *f(x)* and $f'(x)$ through interpretation of graph with focus on gradients. Explanation of the change in gradient, disagreement with Rez's $f'(x)$ graph.	Representational, Intellectual (comprehension, analysis, evaluation)
2 mins	Exploration of the possibility of a hybrid graph.	Representational, Intellectual (analysis, synthesis)
	Discussion of relative magnitude of the gradient at two points.	Intellectual (analysis), Conceptual, Representational
2 mins	Require links between *f(x)* and $f''(x)$.	Intellectual (comprehension)
	Gradient of the gradient discussed.	Linguistic, Intellectual
	Can a negative gradient be increasing?	**Discovered Complexity*
	Tony: '$f''(x)$ gives curve of graph'. No response from other students.	Conceptual, Intellectual (synthesis)

Students in Group 2 appeared to develop misconceptions during the individual work they undertook before collaborative work and retain these during collaborative work. These misconceptions were evident in student responses to questions, student explanations of diagrams, dialogue in the subsequent collaborative session and responses to tests. The development of these misconceptions affected the commonality of background achieved by the students through participation in the Gradient Investigation.

Group 1 discovered more complexities, maintained a higher level of engagement in the task, and developed an understanding of a greater number of concepts. This group continually worked just above their present level of mathematical understanding, moving through their individual *Zones of Proximal Development* (Vygotsky in Wertsch, 1984) as they brainstormed to create new mathematical ideas together. Group 2 discovered only one complexity (Table 4) and generally worked at a level below the present mathematical understanding of one or more group members. Rather than building new ideas together, there were frequent demands for further explanation and challenges of other student's ideas.

Discussion and Conclusions

It is a major proposition to emerge from this research that the process of recognising and exploring *discovered complexities* in a task is a fundamentally constructive and connective activity that inevitably promotes student awareness of the interrelationship of task components and maximises a form of conceptual connectedness. The discovery of complexities is idiosyncratic to the particular group; students do not necessarily discover the same complexities as they solve a problem. A task that provides opportunities for discovered complexities has the potential to optimise student learning but the use of such a task is not sufficient to ensure complexities are discovered.

The use of a task that provides the opportunity for students to discover complexities in a learning environment like Class Collaboration supports and encourages student autonomy and promotes mathematical exploration. This autonomy makes the actual direction of the groups' exploratory activities more difficult to predict. The uncertainty in the direction a group might explore and the differences in the mathematical experiences of each collaborative group create a new set of challenges. Educators could support the implementation of such an approach by addressing issues such as: (a) how to plan a curriculum; (b) how to deliver this curriculum within the time constraints of the school; and (c) how to improve some teacher's perceived lack of capacity to keep pace with the developing ideas within the different collaborative groups.

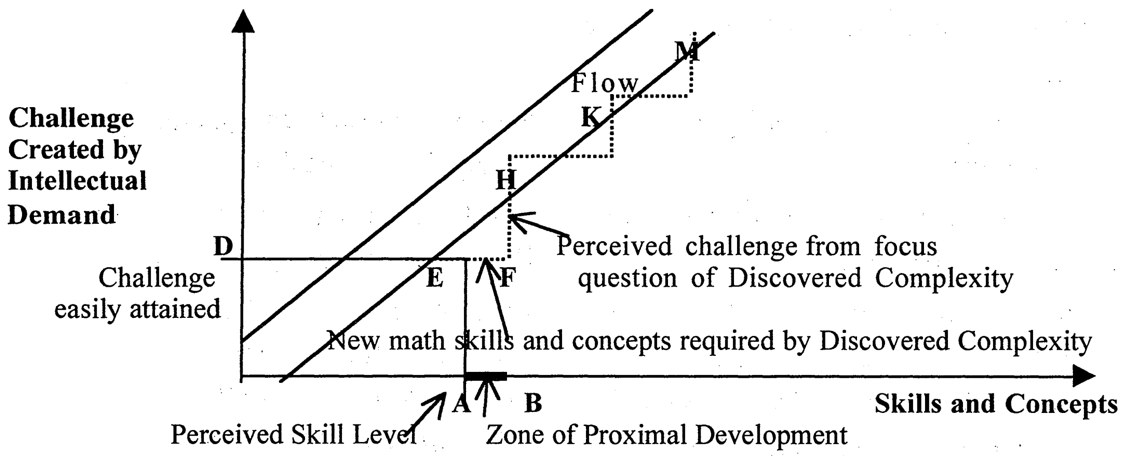

Figure 1. Explanation of sustained engagement integrating Discovered Complexities and Zone of Proximal Development into Csikszentmihalyi's representation of optimal conditions (1992, p. 259).

Tasks providing the opportunity for discovered complexities contain a dynamic aspect that could explain the sustained engagement of members of a collaborative group. An extension of M. Csikszentmihalyi and I. Csikszentmihalyi's (1992, p. 259) schematic representation integrates flow, the zone of proximal development and discovered complexity (Figure 1). The horizontal axis indicates the student's perceived level of skills and concept development and displays the student's present potential for new learning. The vertical axis represents the intellectual challenge the student presently perceives. A student, positioned at **E**, who perceives their skills and concepts level is **A** and the challenge they can easily attain is **D,** is not in flow (between the parallel lines) because this student is working at their present skill level and the challenge is easily within reach. The discovery of a new complexity would position this student in flow at **H** as new mathematical skills and concepts are required (horizontal distance **EF**) and an appropriate challenge is present (vertical distance **EH**). The progressive discovery and familiarisation with complexities sustains flow by moving the student from **H** to **K** to **M**.

The small number and the specificity of the groups in this study would suggest the results should not be generalised without further investigation. Research by Brown (1994) and Tang (1993) with Grade 2 science students and first year tertiary physiotherapy students, respectively, supports the association between student autonomy, student engagement and learning gains. Further research could utilise *discovered complexity* as an analytic tool to explore student response to unfamiliar challenging problems within a broader curriculum context using a variety of tasks, students representative of a wider range of Abilities, and collaborative groups composed of a range of gender proportions. Such a study could extend our understanding of how to create student engagement whilst maximising conceptual development.[1]

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